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AN ANALYSIS OF LIGHT BUYER'S ECONOMIC ORDER MODEL UNDER TRADE CREDIT

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Buyer's inventory policy is related to the credit line and credit period offered by the supplier. A mathematical inventory model of a light buyer is formulated under trade credit and its optimal solution can be determined by five financial indices. On comparing these five financial indices in size, the economic order quantity can be decided. Financial managers can make reference to this property when they make the investment and financing policies of working capital.

Keywords: Inventory, light buyer, trade credit, discounted cash flow.

1. Introduction

Trade credit is composed of credit period and credit line. The application of trade credit has become widespread in the development of economy. Trade credit was regarded as a means of a supplier's marketing strategy. Ashton (1987) points out that trade credit is to lower the effective price of commodity by means of the hidden discount (or called implicit discount). Once a buyer asks for purchasing on credit from the supplier, the supplier will examine the buyer's credit rating through the credit evaluation process and then the credit line and credit period will be determined. The worse the level is, the less the credit line is and the shorter the credit period is. The buyer will even be asked to pay in cash when his credit rating is very poor. Accordingly, trade credit means implicitly a saving of cost and should be considered in the account of inventory cost.

In practice, the supplier often makes use of credit period and credit line to promote his commodities. But, these two factors are always neglected in a traditional economic order quantity model. In order to expand the application of economic order quantity model, many scholars has been trying to improve the defect of oversimplification. Bregman (1992), Carlson

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et al. (1996), Chand and Ward (1987), Chapman *et al.* (1984), Jaggi and Aggarwal (1994), and Shinn *et al.* (1996) only investigated the influence of the credit period on the economic order quantity. Wilson (1991) just investigated the influence of the credit line on the economic order quantity. Although Goyal (1985) was the exceptional one who considered these two factors simultaneously in the model, his research results were on the assumption that the buyer's unit capital cost equals to the return rate of investment opportunity. In fact, the supplier gives the buyer different credit line and credit period according to the buyer's credit rating. The credit line was transformed into the length of usable period during which the credit line is in the duration of consumed commodities by the demand amount per unit time (demand quantity per unit time multiplied by unit purchase price). Since usable credit period is not necessarily equal to the length of their usable period in which the credit line is in the duration of consumed commodities, we embedded these characters into the mathematical model to discuss the effects on the economic order quantity in this paper. The view between this paper and the other one is different.

According to the concept of usage rate segmentation, Kolter *et al.* (1996) point out that suppliers can segment the market into heavy users, light users and nonusers. Those who need a large quantity per unit time are called heavy buyers and those who need a small quantity per unit time are called light buyers. Since buyers have their own specific consumer's behaviour, it is necessary for the supplier to receive the response of marketing strategies to the consumer. Generally speaking, light buyers can only passively accept the credit period and the credit line given by the supplier. In this paper, a supplier's light (heavy or regular) buyer is defined as those who need a small quantity per unit time, which makes their usable credit period shorter (longer or equal to) than the length of their usable period during which the credit line is in the duration of consumed commodities. From a light buyer's standing, this paper examines how he should make his inventory policy properly under trade credit by the concept of discounted cash flow. The results from this research give financial manager a hint at the investment and financing policies of working capital.

2. Notations and Assumptions

2.1. Notations

α = Buyer's demand quantity per unit time

r = Buyer's capital cost

$\bar{X}$ = Credit line offered by the supplier

S = Setup cost of each order placed by the buyer

h = Holding cost per item per unit time

$\bar{t}$ = Credit period offered by the supplier

p = Buyer's purchase price per unit

$\dfrac{\bar{X}}{ap}$ = The length of period when credit line is in the duration of consumed commodities

q = Buyer's order quantity

$x \odot y$ = min $\{x,\ y\}$

$[x]^+$ = max $\{0, x\}$

2.2. Assumptions

a. The buyer will draw a check to the supplier immediately when he/she receives commodities. The nominal account of check will not exceed $\bar{X}$ and the payment date will not exceed $\bar{t}$.

b. Setup cost should be paid by cash when commodities are received.

c. The buyer sells the commodities by cash.

d. The length of the buyer's usable credit period should be shorter than the length of period when the credit line is in the duration of consumed commodities, i.e., $\bar{t} < \dfrac{\bar{X}}{ap}$.

3. Mathematical Model

The corporation value can be evaluated by the present value of cash inflow streams in the future as the field of financial management. The goal of inventory investment is to maximize the present value of cash inflow streams too. Since the time value of money should be embedded into the account of inventory cost, thus the discounted cash flow concept is often used to formulate the mathematical inventory model such as Carlson *et al.* (1996), Grubbstrom (1980), and Jaggi and Aggarwal (1994). The buyer's wealth increases from the sale of commodities and therefore the present value of their wealth can be regarded as a difference between the present value of cash inflow stream and the present value of cash outflow stream in the future. Since the buyer's demand quantity for commodity per unit time is specific and the buyer sells the commodities in cash, the present value of cash inflow streams is fixed. Accordingly, maximizing the present value of the buyer's wealth is equivalent to minimizing the value of their cash outflow in the future. Since capital cost invested in inventory is separated from the holding cost of inventory in this paper, the present value of cash outflow covers the following three items: the cash outflow of purchasing cost, the cash outflow of holding cost, and the cash outflow of setup cost.

When the credit line $\bar{X}$ and credit period $\bar{t}$ are given, the amount of cash paid by the buyer is $[pq - \bar{X}]^+$, and the amount of freely financing hold by the buyer is $(pq \odot \bar{X})$. If the length of inventory cycle $\frac{q}{a}$ determined by the buyer is shorter than the length of credit period $\bar{t}$ given by the supplier, the credit period can be represented as $(\frac{q}{a} \odot \bar{t})$ since he does not repay the prior accumulated debts and is not permitted to enjoy another trade credit. Therefore, the present value of purchasing cost per order is $[pq - \bar{X}]^+ + (pq \odot \bar{X})e^{-r(\frac{q}{a} \odot \bar{t})}$.

If the buyer's demand quantity per unit time is a and order quantity is q, the buyer's inventory is $q - at$ at time t. If the holding cost of inventory per unit is h, the holding cost of inventory at time t is $h(q-at)$. Accordingly, the present value of holding cost of inventory is $\int_0^{\frac{q}{a}} h(q-at)e^{-rt}dt$ within the inventory cycle $[0, \frac{q}{a}]$.

As we discussed above, the present value $PV(q)$ of total inventory cost within the inventory cycle $[0, \frac{q}{a}]$ is

$$\begin{aligned} PV(q) &= \text{the present value of setup cost} + \text{the present value of holding} \\ &\quad\ \text{cost} + \text{the present value of purchasing cost} \\ &= S + \int_0^{\frac{q}{a}} h(q-at)e^{-rt}dt + [pq - \bar{X}]^+ + (pq \odot \bar{X})e^{-r(\frac{q}{a} \odot \bar{t})} \\ &= S + \frac{hq}{r} - \frac{ah}{r^2} + \frac{ah}{r^2}e^{-r\frac{q}{a}} + [pq - \bar{X}]^+ + (pq \odot \bar{X})e^{-r(\frac{q}{a} \odot \bar{t})} \qquad (3.1) \end{aligned}$$

If the length of inventory cycle is $\frac{q}{a}$, the order points are $0, \frac{q}{a}, 2\frac{q}{a}, 3\frac{q}{a}, \cdots$, then the present value $TPV(q)$ of total cost with order quantity q in the future is

$$TPV(q) = PV(q) + PV(q)e^{-r\frac{q}{a}} + PV(q)e^{-2r\frac{q}{a}} + \cdots$$
$$= \frac{PV(q)}{1 - e^{-r\frac{q}{a}}} \tag{3.2}$$

Based on the concept of present value of perpetuity, the present value of total cost can be converted to equivalent annual annuity $AAN(q)$ as the following:

$$AAN(q) = TPV(q) * r$$
$$= \frac{PV(q)}{1 - e^{-r\frac{q}{a}}} * r$$
$$= r\{\frac{S + \frac{hq}{r} + [pq - \bar{X}]^+ + (pq \odot \bar{X})e^{-r(\frac{q}{a} \odot \bar{t})}}{1 - e^{-r\frac{q}{a}}}\} - \frac{ah}{r} \tag{3.3}$$

Given $(\bar{X}, \bar{t})$, we will determine the order quantity q by setting up a mathematical model to minimize the equivalent annual annuity as the following:

$$\text{Model 1} : \min_{q \geq 0} AAN(q) = r\{\frac{S + \frac{hq}{r} + [pq - \bar{X}]^+ + (pq \odot \bar{X})e^{-r(\frac{q}{a} \odot \bar{t})}}{1 - e^{-r\frac{q}{a}}}\} - \frac{ah}{r} \tag{3.4}$$

4. Optimal Solution

From (3.4), we can get

$$AAN'(q) = \frac{r}{(1 - e^{-r\frac{q}{a}})(e^{r\frac{q}{a}} - 1)} \times F(q), \quad q \neq a\bar{t}, \ \frac{\bar{X}}{p} \tag{4.1}$$

in which $F(q)$ is defined as:

when $q < a\bar{t}$,

$$F(q) = F_1(q)$$
$$= \frac{h}{r}(e^{r\frac{q}{q}} - r\frac{q}{a} - 1) - p(e^{-r\frac{q}{a}} - 1 + r\frac{q}{a}) - \frac{rS}{a} \tag{4.2}$$

when $a\bar{t} \leq q < \frac{\bar{X}}{p}$,

$$F(q) = F_2(q) = (\frac{h}{r} + pe^{-r\bar{t}})(e^{r\frac{q}{a}} - r\frac{q}{a} - 1) - \frac{rS}{a} \tag{4.3}$$

when $q \geq \frac{\bar{X}}{p}$,

$$F(q) = F_3(q) = (\frac{h}{r} + p)(e^{r\frac{q}{a}} - r\frac{q}{a} - 1) - \frac{rS}{a} + r\frac{\bar{X}}{a}(1 - e^{-r\bar{t}}) \tag{4.4}$$

By (4.1),

$$AAN'(q) > 0 \text{ if and only if } F(q) > 0 \tag{4.5}$$

From (4.2), (4.3) and (4.4), we get

$$F(a\bar{t}^-) = \frac{h}{r}(e^{r\bar{t}} - 1 - r\bar{t}) - p(e^{-r\bar{t}} - 1 + r\bar{t}) - \frac{rS}{a} \tag{4.6}$$

$$F(a\bar{t}^+) = (\frac{h}{r} + pe^{-r\bar{t}})(e^{r\bar{t}} - 1 - r\bar{t}) - \frac{rS}{a} \tag{4.7}$$

$$F(\frac{\bar{X}^-}{p}) = (\frac{h}{r} + pe^{-r\bar{t}})(e^{\frac{r\bar{X}}{ap}} - 1 - \frac{r\bar{X}}{ap}) - \frac{rS}{a} \tag{4.8}$$

$$F(\frac{\bar{X}^+}{p}) = (\frac{h}{r} + p)(e^{\frac{r\bar{X}}{ap}} - 1 - \frac{r\bar{X}}{ap}) - \frac{rS}{a} + r\frac{\bar{X}}{a}(1 - e^{-r\bar{t}}) \tag{4.9}$$

Since $a\bar{t} < \frac{\bar{X}}{p}$, and if $x \geq 0$, both $(e^x - 1 - x) \geq 0$ and $(e^{-x} - 1 + x) \geq 0$ hold. Therefore, by (4.6), (4.7), (4.8) and (4.9), we get

$$F(a\bar{t}^-) \leq F(a\bar{t}^+) \leq F(\frac{\bar{X}^-}{p}) \leq F(\frac{\bar{X}^+}{p}) \tag{4.10}$$

Proposition 1. *Supposed q^* is the optimal solution of model 1*

Case A : if $\frac{rS}{a} < \frac{h}{r}(e^{r\bar{t}} - 1 - r\bar{t}) + p(1 - r\bar{t} - e^{-r\bar{t}})$, then $AAN(q^*) = \min_{q<a\bar{t}} AAN(q)$ and q^* satisfies $\frac{h}{r}(e^{r\frac{q^*}{a}} - r\frac{q^*}{a} - 1) - p(e^{-r\frac{q^*}{a}} - 1 + r\frac{q^*}{a}) = \frac{rS}{a}$.

Case B : if $\frac{h}{r}(e^{r\bar{t}} - 1 - r\bar{t}) + p(1 - r\bar{t} - e^{-r\bar{t}}) \leq \frac{rS}{a} < (\frac{h}{r} + pe^{-r\bar{t}})(e^{r\bar{t}} - 1 - r\bar{t})$, then $q^* = a\bar{t}$.

Case C : *if* $(\frac{h}{r}+pe^{-r\bar{t}})(e^{r\bar{t}}-1-r\bar{t}) \le \frac{rS}{a} < (\frac{h}{r}+pe^{-r\bar{t}})(e^{\frac{r\bar{X}}{ap}}-1-\frac{r\bar{X}}{ap})$, *then* $AAN(q^*) = \min_{a\bar{t}\le q<\frac{\bar{X}}{p}} AAN(q)$ *and* q^* *satisfies* $(\frac{h}{r}+pe^{-r\bar{t}})(e^{r\frac{q^*}{a}} - r\frac{q^*}{a}-1) = \frac{rS}{a}$.

Case D : *if* $(\frac{h}{r}+pe^{-r\bar{t}})(e^{\frac{r\bar{X}}{ap}}-1-\frac{r\bar{X}}{ap}) \le \frac{rS}{d} < (\frac{h}{r}+p)(e^{\frac{r\bar{X}}{ap}}-1-\frac{r\bar{X}}{ad}) + r\frac{\bar{X}}{a}(1-e^{-r\bar{t}})$, *then* $q^* = \frac{\bar{X}}{p}$.

Case E : *if* $(\frac{h}{r}+p)(e^{\frac{r\bar{X}}{ap}}-1-\frac{r\bar{X}}{ap}) + r\frac{\bar{X}}{a}(1-e^{-r\bar{t}}) \le \frac{rS}{a}$, *then* $AAN(q^*) = \min_{q\ge\frac{\bar{X}}{p}} AAN(q)$ *and* q^* *satisfies* $(\frac{h}{r}+p)(e^{r\frac{q^*}{q}} - r\frac{q^*}{a}-1) + r\frac{\bar{X}}{a}(1-e^{-r\bar{t}}) = \frac{rS}{a}$.

Proof. By (3.3),

$AAN(q)$ is a continuous function in the interval $(0,\infty)$ and

$$\lim_{q\to 0^+} AAN(q) = \infty \tag{4.11}$$

By (4.3), $F_2(0) < 0$, $F_2(\infty) > 0$ and $F_2'(q) = (\frac{h}{r}+pe^{-r\bar{t}})(\frac{r}{a}e^{r\frac{q}{a}} - \frac{r}{a}) > 0$, $\forall q > 0$. Hence

there exists an unique positive real number q satisfying $F_2(q) = 0$ (4.12)

By (4.4), $F_3(\infty) > 0$, $F_3'(q) = (\frac{h}{r}+p)(\frac{r}{a}e^{r\frac{q}{a}} - \frac{r}{a}) > 0$, $\forall q > 0$, and the necessary and sufficient conditions of $F_3(0) < 0$ is $\bar{X}(1-e^{-r\bar{t}}) < S$. Hence

if $\bar{X}(1-e^{-r\bar{t}}) < S$, there exists an unique positive real number q satisfying $F_3(q) = 0$; otherwise $F_3(q) > 0$, $\forall q > 0$ (4.13)

By (4.2), $F_1(0) < 0$, $F_1(\infty) = \infty$, and (i) if $\ln\frac{rp}{h} < 0$, $F_1(q)$ is a strictly increasing function in interval $(0,\infty)$. (ii) if $\ln\frac{rp}{h} > 0$, the function $F_1(q)$ is minimal at the point $q = \frac{a}{r}\ln\frac{rp}{h}$, and $F_1(q)$ is a strictly increasing function in interval $[\frac{a}{r}\ln\frac{rp}{h},\infty)$, but a strictly decreasing function in the interval $(0,\frac{a}{r}\ln\frac{rp}{h}]$. From (i) and (ii),

there exists an unique real number q, $q \in ([\frac{a}{r}\ln\frac{rp}{h}]^+, \infty)$, satisfying $F_1(q) = 0$ (4.14)

Proof of Case 1 : Supposed $F(a\bar{t}^-) > 0$ and by (4.10),

$$0 < F(a\bar{t}^-) \le F(a\bar{t}^+) \le F(\frac{\bar{X}^-}{p}) \le F(\frac{\bar{X}^+}{p}) \tag{4.15}$$

Based on (4.13) and (4.14), we get $F(q) = F_3(q) > 0,\ \forall q \geq \frac{\bar{X}}{p}$; therefore by (4.5), we get

$$AAN'(q) > 0,\ \forall q \in [\frac{\bar{X}}{p}, \infty) \tag{4.16}$$

Based on (4.12) and (4.15), we get $F(q) = F_2(q) > 0,\ \forall a\bar{t} \leq q < \frac{\bar{X}}{p}$; therefore by (4.5), we get

$$AAN'(q) > 0,\ \forall q \in [a\bar{t}, \frac{\bar{X}}{p}) \tag{4.17}$$

From (4.11), (4.16) and (4.17),

$$AAN(q^*) = \min_{q \geq 0} AAN(q) = \min_{0<q<a\bar{t}} AAN(q) \tag{4.18}$$

and by (4.2), q^* must satisfy

$$0 = F_1(q^*) = \frac{h}{r}(e^{r\frac{q^*}{a}} - 1 - r\frac{q^*}{a}) - p(e^{-r\frac{q^*}{a}} - 1 + r\frac{q^*}{a}) - \frac{rS}{a}$$

Consequently Case A was verified. Case B to E can be concluded by referring to the above proving process.

In order to expand the application of proposition 1 in practice, we find that there exists a close relationship between the optimal solution and the following five financial indices:

Index 1 : $\frac{h}{r}(e^{r\bar{t}} - 1 - r\bar{t}) + p(1 - r\bar{t} - e^{-r\bar{t}})$

Just within the credit period, the perpetuated holding cost per unit differing in future value between continuous compound interest and simple interest is added to purchase price per unit differing in present value between continuous compound interest and simple interest.

Index 2 : $(\frac{h}{r} + pe^{-r\bar{t}})(e^{-r\bar{t}} - 1 - r\bar{t})$

Just within the credit period, the perpetuated holding cost per unit and discounted purchase price per unit differ in future value between continuous compound interest and simple interest.

Index 3 : $(\frac{h}{r} + pe^{-r\bar{t}})(e^{\frac{r\bar{X}}{ap}} - 1 - \frac{r\bar{X}}{ap})$

Just within the period during which the credit line is in the duration of consumed commodities, the perpetuated holding cost per unit and discounted purchase price per unit differ in future value between continuous compound interest and simple interest.

Index 4 : $(\frac{h}{r}+p)(e^{\frac{e\bar{X}}{ap}}-1-\frac{r\bar{X}}{ap})+r\frac{\bar{X}}{a}(1-e^{-r\bar{t}})$

Just within the period during which credit line is in the duration of consumed commodities, the perpetuated holding cost per unit and purchase price per unit which differ in future value between continuous compound interest and simple interest plus interest revenue incurred from the saving of each commodity under the free-financing of credit line.

Index 5: $\frac{rS}{a}$

The interest on setup cost was shared by each unit commodity per unit time.

We prove that the previous four indices can be arranged in series as the following:

$$\text{Index 1} \leq \text{Index 2} \leq \text{Index 3} \leq \text{Index 4} \tag{4.19}$$

It is also one of the main research results of this paper. On comparing Index 5 with the previous four indices in size, order quantity q^* is decided under all circumstances as shown in Figure 1. For example, Index 5 is left to financial Index 1 as above, the economic order quantity q^* is less the demand quantity within the credit period. If Index 5 is positioned between Index 1 and 2, the economic order quantity is equal to the demand quantity within the credit period. The rest may be inferred by analogy.

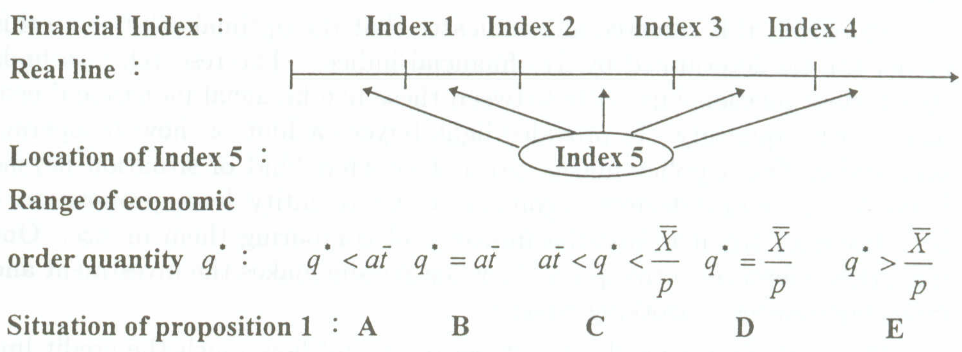

Figure 1. The relations of financial index and economic order quantity

Example 1. Let $a = 500$, $r = 10\%$, $h = 0.1$, $p = 1$, $S = 5000$. By Newton's approximation method ($|\text{error}| \leq 10^{-5}$) the optimal order quantity is shown in Table 1 under the different combination $(\bar{X}, \bar{t})$.

Table 1. The optimal order quantity under $(\bar{X}, \bar{t})$

$(\bar{X}, \bar{t})$	q^*	AAN	Index				
			1	2	3	4	5
(3000, 06)	3296.64	1423.97	0.07	0.34	0.56	1.07	1.25
(3500, 07)	3500.00	1355.68	0.12	0.50	0.78	1.49	1.25
(4000, 08)	4000.00	1308.11	0.18	0.62	1.04	1.99	1.25
(4500, 09)	4368.93	1277.15	0.25	0.79	1.34	2.58	1.25
(4500, 10)	4419.21	1251.65	0.35	0.98	1.31	2.62	1.25

5. Conclusion

When the supplier offers credit period and credit line, the buyer will try his best to make use of these free-financing opportunities. For a buyer whose demand quantity per unit time is not so large that the enjoyable credit period would be shorter than the period that credit line is in the duration of consumed commodities, we define this buyer as a light buyer. Taking a light buyer's standing, how to formulate an optimal inventory policy is always a practical problem in order to minimize inventory cost under the trade credit. Based on these complex situations, one main result of this research is to set up a mathematical model which can be concretely discussed.

Moreover, this research also indicates that the optimal solution to this model can be determined by five financial indices. The research also finds that a close relationship exists between these five financial indices and economic order quantity. It provides light buyers a hint on how to operate practically. For a policy-maker, no matter what kind of situation he/she faces, he/she can determine economic order quantity from proposition 1 by estimating the five financial indices and comparing them in size. One can make reference to this property when he/she makes the investment and financing policies of working capital.

To decide how long the credit period is and how much the credit line is for the supplier may be the possible direction in the future research.

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